
Student Number

Barker College
125
1890-2015

Mathematics

2015
TRIAL
HIGHER SCHOOL CERTIFICATE

AM Friday 31 July

Section 1 – Multiple Choice

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

(A) ○ (B) ○ (C) ○ (D) ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

(A) ● (B) ⊗ (C) ○ (D) ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.

correct

(A) ● (B) ⊗ (C) ○ (D) ○

Start Here →

1. A○ B○ C○ D○

2. A○ B○ C○ D○

3. A○ B○ C○ D○

4. A○ B○ C○ D○

5. A○ B○ C○ D○

6. A○ B○ C○ D○

7. A○ B○ C○ D○

8. A○ B○ C○ D○

9. A○ B○ C○ D○

10. A○ B○ C○ D○

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Staff Involved:
- AJD*
- LMD
- VAB
- ARM
- GIC
- PJR*
- DZP
- GPF
- WMD
- JGD

AM Friday 31 July

120 copies

General Instructions
- Reading time - 5 minutes
- Working time - 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Write your Barker Student Number on all pages of your solutions
- In Questions 11 – 16, show all relevant mathematical reasoning and/or calculations

Total marks – 100

Section I **Pages 2 - 3**

10 marks
- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II **Pages 5 - 12**

90 marks
- Attempt Questions 11 - 16
- Allow about 2 hours and 45 minutes for this section

Section 1 - Multiple Choice (10 marks)

Attempt Questions 1 - 10

Use the multiple-choice answer sheet for Questions 1 - 10

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1. $\log_4 2 =$

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 4

2. Solve $|3 - x| \geq 6$

(A) $x \geq 3$ or $x \leq 9$ (B) $x \leq -3$ or $x \geq 9$

(C) $x \leq -3$ or $x \leq 9$ (D) $-3 \leq x \leq 9$

3. The limiting sum of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \ldots$ is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $1\frac{1}{2}$ (D) 2

4. If $5\sqrt{2} - \sqrt{8} + \sqrt{32} = \sqrt{x}$, the value of x is:

(A) 26 (B) $\sqrt{98}$ (C) 98 (D) 130

5. The derivative of $\cos 2x$ is:

(A) $-2\cos 2x$ (B) $2\cos 2x$ (C) $2\sin 2x$ (D) $-2\sin 2x$

6. Differentiate $\frac{x\sqrt{x}}{x^5}$

(A) $\frac{-7}{2x^4\sqrt{x}}$ (B) $\frac{2\sqrt{x}}{7x^3}$ (C) $\frac{7}{2x^2\sqrt{x}}$ (D) $\frac{-7}{2x^3}$

7. Two-digit numbers are formed from the digits 2, 3, 4, 6 with no repetition of digits allowed.

A two-digit number is then selected at random.

What is the probability that the number is prime?

(A) $\frac{1}{12}$ (B) $\frac{1}{8}$ (C) $\frac{1}{6}$ (D) $\frac{5}{12}$

8. The solution of the inequality $x^2 - 6x + 8 \leq 0$ is:

(A) $x \geq 4$ or $x \leq 2$

(B) $2 \leq x \leq 4$

(C) $2 < x < 4$

(D) $x \leq 4$ or $x \geq 2$

9. The quadratic equation with roots k and $3k$ is:

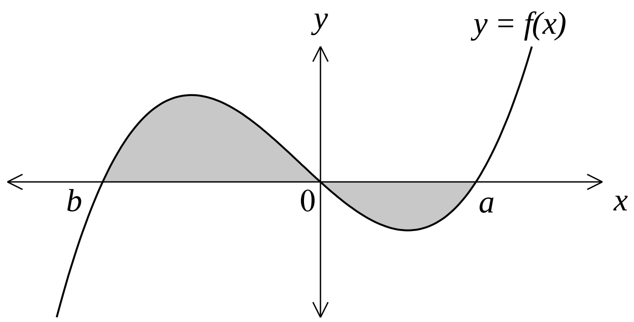
(A) $(x-k)(x-3) = 0$

(B) $x^2 - 4kx + 3k^2 = 0$

(C) $x^2 + 4kx + 3k^2 = 0$

(D) $x^2 + 3k^2x - 4k = 0$

10. Which of the following correctly finds the shaded area in this diagram?



(A) $\int_b^a f(x)\,dx$

(B) $\left| \int_b^a f(x)\,dx \right|$

(C) $\left| \int_0^a f(x)\,dx \right| + \int_b^0 f(x)\,dx$

(D) $\int_0^a f(x)\,dx + \left| \int_b^0 f(x)\,dx \right|$

End of Section I

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Section II – Extended Response (90 marks)

Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question on a separate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) [START A NEW BOOKLET] **Marks**

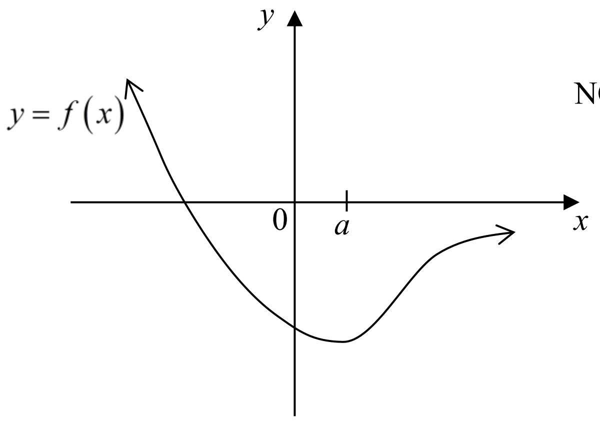
(a) Factorise fully: $4x^3 - 32$ **2**

(b) Find: $\lim_{x \to 0} \dfrac{x^2 - 4x}{2x}$ **1**

(c) Find the integers a and b such that: $\dfrac{4\sqrt{3}}{\sqrt{7} + \sqrt{3}} = a + b\sqrt{21}$ **2**

(d) Copy the following graph of $y = f(x)$ into your answer booklet. **2**

Hence, clearly sketch and label $y = f'(x)$



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Question 11 continues on page 6

Question 11 (continued)

(e) Solve for x: $4^x - 5 \times 2^x + 4 = 0$ **2**

(f) Find the domain of the function $f(x) = \sqrt{x^2 + x - 6}$ **2**

(g) (i) Use Simpson's Rule with three function values to find an approximation to the area under the curve $y = \frac{1}{x}$ between $x = m$ and $x = 3m$, where $m > 0$ **2**

(ii) Hence, using integration, show that $\log_e 3 \approx \frac{10}{9}$ **2**

End of Question 11

Question 12 (15 marks) [START A NEW BOOKLET] **Marks**

(a) Evaluate the arithmetic series $180 + 165 + 150 + \ldots -15 -30$ **2**

(b) (i) Sketch the curve $y = \dfrac{3}{x-1}$ showing any intercepts. **2**

(ii) State the domain and range of $y = \dfrac{3}{x-1}$ **2**

(iii) Hence, evaluate $\displaystyle\int_2^4 \frac{3}{x-1}\,dx$ **2**

(c) Find

(i) $\int 3e^{7x}\ dx$ **1**

(ii) $\int \tan^2 x\ dx$ **2**

(d) The roots of the equation $x^2 + 4x + 1 = 0$ are α and β. Find:

(i) $\alpha + \beta$ **1**

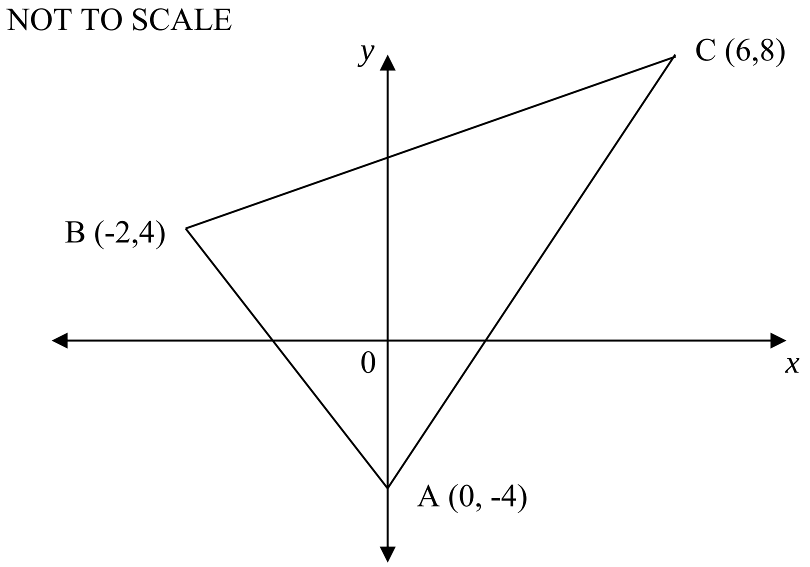
(ii) $\alpha\beta$ **1**

(iii) $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$ **2**

End of Question 12

Question 13 (15 marks) [START A NEW BOOKLET] **Marks**

(a)



In the diagram the coordinates of the points are A(0, -4), B(-2, 4) and C(6, 8).

(i) Show the equation of line BC is $x - 2y + 10 = 0$ **2**

(ii) Find the perpendicular distance from A to the line BC **2**

(iii) Find point D such that ABCD is a parallelogram. **1**

(iv) Hence, find the area of the parallelogram ABCD **2**

Question 13 continues on page 9

Question 13 (continued)

(b) Differentiate with respect to x.

(i) $y = \sqrt{7-3x^2}$ **2**

(ii) $y = \dfrac{3x}{\ln x}$ **2**

(c) The equation of the tangent to the curve $f(x) = 2e^x\left(x^2+1\right)$ at the point (0, 2) **4**

is given by $y = ax + b$

Evaluate a and b.

End of Question 13

Question 14 (15 marks) [START A NEW BOOKLET] **Marks**

(a) Sketch the parabola

$(y+2)^2 = -4(x+3)$ clearly indicating the focus and directrix. **2**

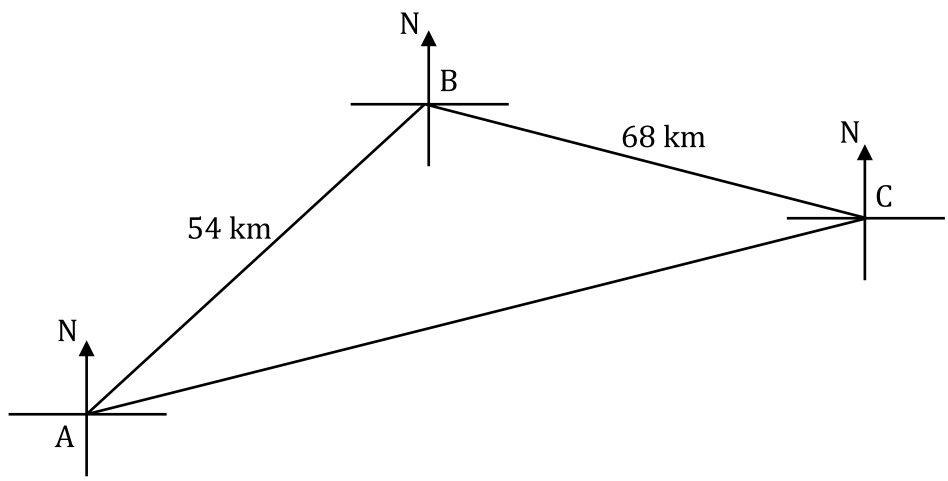
(b) A deck contains six cards labelled the numbers 0, 1, 2, 2, 3 and 3 respectively.

Ann draws two cards at random without replacement.

(i) Find the probability that the sum of the two cards equals 5 **1**

(ii) Find the probability that the sum of the two cards is less than five **2**

(c)



A plane flies 54 km from *A* to *B* on a bearing of 055°.

The plane then continues onto *C*, flying a distance of 68 km on a bearing of 105°.

(i) Copy the diagram into your answer booklet and use this diagram to show why $\angle ABC = 130^\circ$ **1**

(ii) Find the distance *CA* (to nearest 0.1 km). **2**

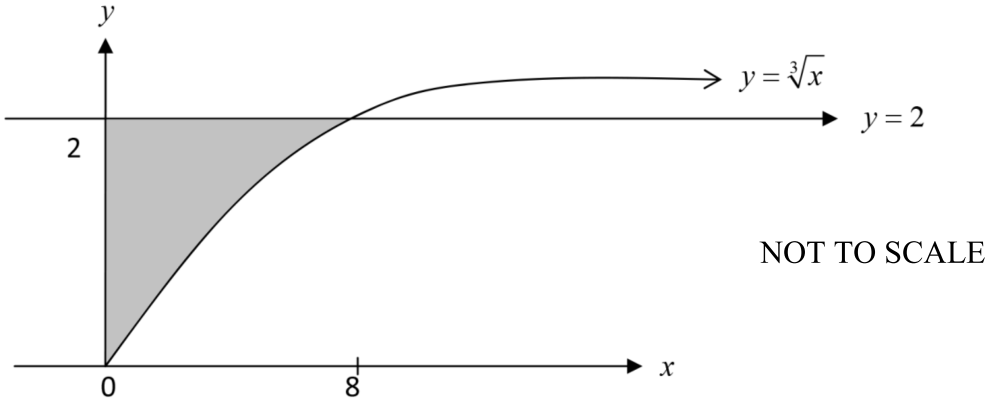
(iii) Hence, calculate the size of $\angle BAC$ (to nearest degree). **2**

(iv) Hence, or otherwise, find the bearing of *A* from *C* (to nearest degree). **1**

Question 14 continues on page 11

Question 14 (continued)

(d)



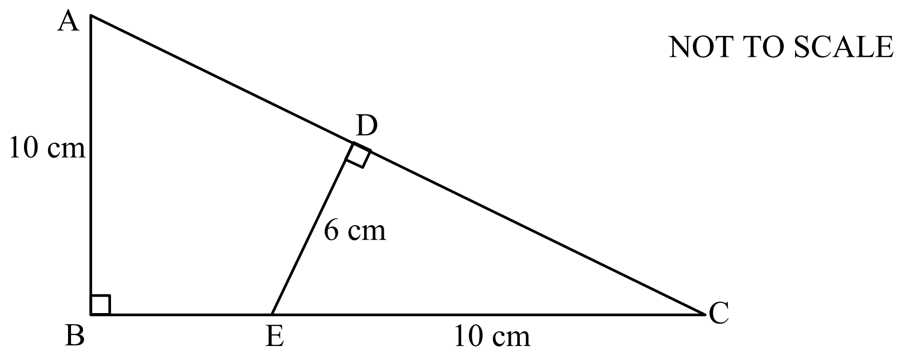
The above diagram shows the region bounded by the curve $y = \sqrt[3]{x}$, the y axis and the line $y = 2$.

Given that the point of intersection of $y = \sqrt[3]{x}$ and $y = 2$ is (8, 2), find the exact volume of the solid formed when the region shown is rotated about the x-axis. **4**

End of Question 14

Question 15 (15 marks) [START A NEW BOOKLET] **Marks**

(a) Triangles *ABC* and *CDE* are right angled at *B* and *D* respectively as shown in the diagram.



(i) Prove that $\triangle ABC \,|||\, \triangle CDE$ **2**

(ii) If $AB = EC = 10$ cm and $DE = 6$ cm, determine the length of *AC*. **2**

(b) Over the interval $a \le x \le b$, a curve $y = f(x)$ has the following properties: **2**

$$f(a) < 0 \;,\quad f'(x) > 0 \quad \text{and} \quad f''(x) < 0$$

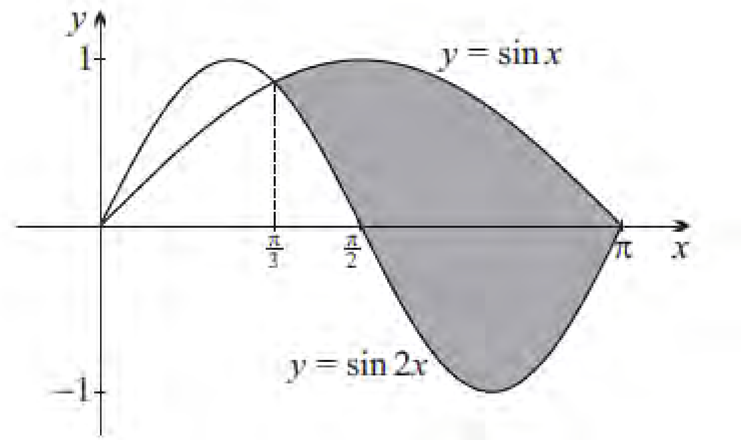
Draw this section of the curve $y = f(x)$ illustrating all of the above information.

(c) Starting with the number 3, Mike multiplies this by a whole number n. **3**
After writing down the result, he then multiplies this answer by n again and continues multiplying until he reaches 3072. In total, he multiplies by n ten times.
He then adds up all his answers (including the original 3).
Find the total Mike calculated.

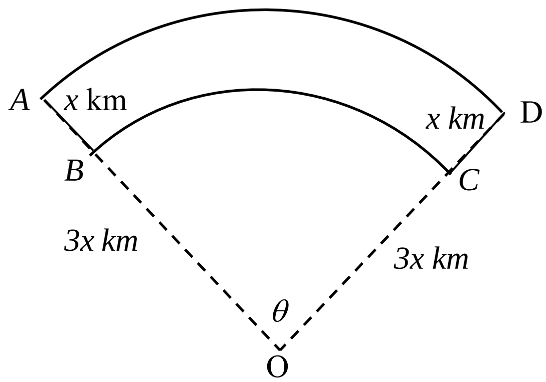
Question 15 continues on page 13

(d) The diagram below shows the curves $y = \sin 2x$ and $y = \sin x$ for $0 \le x \le \pi$ which intersect at the values $x = 0$, $x = \dfrac{\pi}{3}$ and $x = \pi$. **3**

Find the exact area of the shaded region bounded by these two curves.



(e) Four towns *A*, *B*, *C* and *D* are joined by roads that are either straight or arcs of concentric circles with centre at *O*. Town *B* and *C* are 3*x* km from *O*. Towns *A* and *D* are both *x* km from *B* and *C* respectively. $\angle AOD = \theta$ radians as shown in the diagram below.



(i) Write an expression in terms of *x* and θ for the length of arc AD **1**

(ii) A salesperson wants to travel from Town A to Town D but must visit town B and C on the way. Write an expression in terms of θ and *x*, for the length of this journey from Town A to Town D. **1**

(iii) Find the value of θ for which the journeys in (i) and (ii) are the same distance. **1**

End of Question 15

Question 16 (15 marks) [START A NEW BOOKLET] **Marks**

(a) The curve $y = f(x)$ has two stationary points at $x = 1$ and $x = a$ **2**

If $f''(x) = 6x - 2$, find the value of a.

(b) Solve the equation: **3**

$2\cos^2 x = 2\sin x\cos x$ where $0 \le x \le 2\pi$

(c) Rachael borrows $\$P$ for an overseas holiday.
This loan plus interest is to be repaid in equal monthly instalments of \$232 over five years.
Interest of 6% p.a. is charged monthly on the balance owing at the start of each month.

Let A_n be the amount owing at the end of n months.

(i) Show that $A_2 = P(1.005)^2 - 232(1 + 1.005)$ **2**

(ii) Prove that $A_n = P(1.005)^n - 46\,400(1.005^n - 1)$ **2**

(iii) Hence, or otherwise, find the amount that Rachael borrowed. **2**

(d) A basketball coach knows with certainty that the number of points scored during a season by a player with shirt number n is given by: **4**

$$P = n^3 - 30n^2 + 225n$$

If the shirt numbers are only whole numbers from 1 to 20 inclusive, which player(s) will score the most points in this season and determine their score(s)?

End of Paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Trial HSC 2015 31/7/15 Mathematics

Q1 a) $\log_4 2 = x$

$4^x = 2$

$2^{2x} = 2^1$

$2x = 1 \quad x = \frac{1}{2}$ A

2) $|3-x| \geq 6$

$3 - x \geq 6$, $-3 + x \geq 6$

$-x \geq 3$, $x \geq 9$

$x \leq -3$ B

3) $a = 1$, $r = -\frac{1}{2}$ $S_\infty = \frac{1}{1 - -\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$ B

4) $5\sqrt{2} - 2\sqrt{2} + 4\sqrt{2} = 7\sqrt{2} = \sqrt{98}$

$\therefore \sqrt{x} = \sqrt{98}$

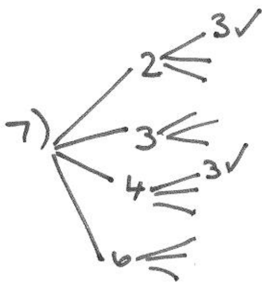
$x = 98$ C

5) $\frac{d}{dx}\cos 2x = -2\sin 2x$ D

6) $\frac{x^1 \times x^{\frac{1}{2}}}{x^5} = x^{-\frac{7}{2}}$

$\frac{dy}{dx} = -\frac{7}{2}x^{-\frac{9}{2}} = \frac{-7}{2\sqrt{x^9}}$

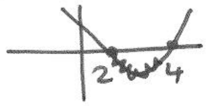
$= \frac{-7}{2x^4\sqrt{x}}$ A

7) 

$4 \times 3 = 12$ combinations

$\frac{2}{12} = \frac{1}{6}$ C

8) $(x-2)(x-4) \leq 0$ B



$2 \leq x \leq 4$

9) $(x-k)(x-3k) = 0$ B

$x^2 - 4kx + 3k^2 = 0$

10) C

11) a) $4(x^3 - 8) = 4(x-2)(x^2+2x+4)$

b) $\lim_{x\to 0} \frac{x(x-4)}{2x} = \lim_{x \to 0}\frac{x-4}{2}$

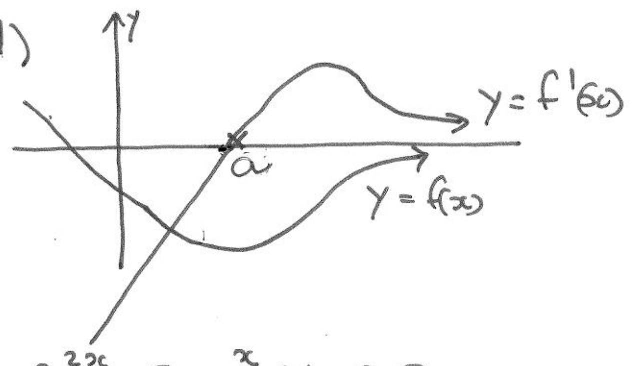
$= \frac{0-4}{2} = -2$

11 c) $\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} = \frac{4\sqrt{21} - 4\times 3}{7-3}$

$= \frac{4(\sqrt{21}-3)}{4}$

$= \sqrt{21} - 3$

$\therefore a = -3$, $b = 1$

d) 

e) $2^{2x} - 5 \times 2^x + 4 = 0$

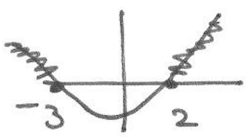
$m^2 - 5m + 4 = 0$

$(m-1)(m-4) = 0 \quad m = 1, 4$

$2^x = 2^0, 2^2 \quad x = 0, 2$

f) $x^2 + x - 6 \geq 0$

$(x+3)(x-2) \geq 0$



$x \leq -3$ or $x \geq 2$

g) i)

x	m	$2m$	$3m$
y	$\frac{1}{m}$	$\frac{1}{2m}$	$\frac{1}{3m}$

$A = \frac{m}{3}\left(\frac{1}{m} + \frac{1}{3m} + \frac{4}{2m}\right)$

$= \frac{m}{3} \times \frac{10}{3m} = \frac{10}{9}\ u^2$

ii) $\int_m^{3m} \frac{1}{x}dx = \ln x \Big]_m^{3m}$

$= \ln 3m - \ln m$

$= \ln(3m/m)$

$= \log_e 3 \approx 10/9$

12) a) $a = 180 \quad d = -15$

$l = a + (n-1)d$

$-30 = 180 + (n-1) \times -15$

$-210 = -15n + 15$

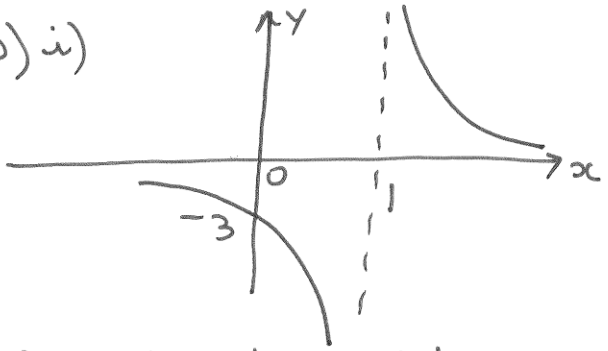
$-15n = -225$

$n = 15$

$S_{15} = \frac{15}{2}(180 - 30)$

$= 1125$

b) i)



ii) x all real $x \neq 1$

y all real $y \neq 0$

iii) $\int_2^4 \frac{3}{x-1}\,dx = 3\ln(x-1)\Big]_2^4$

$= 3(\ln 3 - \ln 1)$

$= 3\ln 3$

c) i) $\frac{3}{7}\int 7e^{7x}\,dx = \frac{3}{7}e^{7x} + C$

ii) $\int \tan^2 x\,dx = \int \sec^2 x - 1\,dx$

$= \tan x - x + C$

d) $a = 1$, $b = 4$, $c = 1$

i) $\alpha + \beta = \frac{-4}{1} = -4$

ii) $\alpha\beta = \frac{1}{1} = 1$

iii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$

$= \frac{16-2}{1}$

$= 14$

13) i) $m_{BC} = \frac{8-4}{6+2} = \frac{4}{8} = \frac{1}{2}$

$y - 4 = \frac{1}{2}(x+2)$

$2y - 8 = x + 2$

$x - 2y + 10 = 0$

ii) $(0, -4)$

$d = \left| \frac{1(0) - 2(-4) + 10}{\sqrt{1^2 + 2^2}} \right|$

$= \left| \frac{18}{\sqrt{5}} \right| = \frac{18}{\sqrt{5}}\ u^2$

iii) $M_{AC} = (3, 2)$

(-2, 4) (3, 2) D(x, y)

$D\left(\frac{-2+x}{2}, \frac{4+y}{2}\right) = (3, 2)$

$\frac{-2+x}{2} = 3$ $\quad$ $\frac{4+y}{2} = 2$

$x = 8$ $\quad$ $y = 0$

$D(8, 0)$

iv) Area $= b \times ph$

$BC = \sqrt{8^2 + 4^2}$

$= \sqrt{80}$

$= 4\sqrt{5}$

$= 4\sqrt{5} \times \frac{18}{\sqrt{5}}$

$= 72\ u^2$

b) i) $y = (7 - 3x^2)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(7 - 3x^2)^{-\frac{1}{2}} \times -6x$

$= -3x(7 - 3x^2)^{-\frac{1}{2}}$

ii) $\frac{dy}{dx} = \frac{\ln x \times 3 - 3x \times \frac{1}{x}}{(\ln x)^2}$

$= \frac{3\ln x - 3}{(\ln x)^2}$

13 c) $f(x) = 2e^{x}(x^2+1)$

$f'(x) = 2e^{x}(x^2+1) + 2e^{x} \times 2x$

$= 2e^{x}(x^2+2x+1)$

$f'(0) = 2(1) = 2$

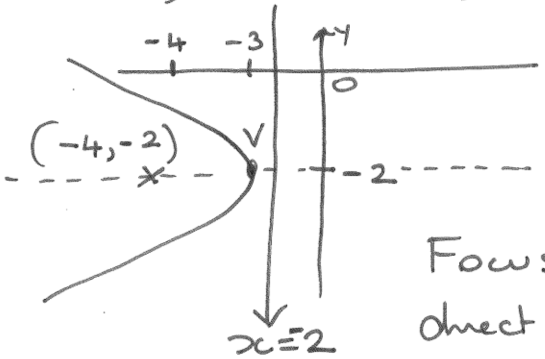
$x=0 \quad f(0) = 2(1) = 2$

$y-2 = 2(x-0)$

$y = 2x+2$

$\therefore a = 2, \quad b = 2$

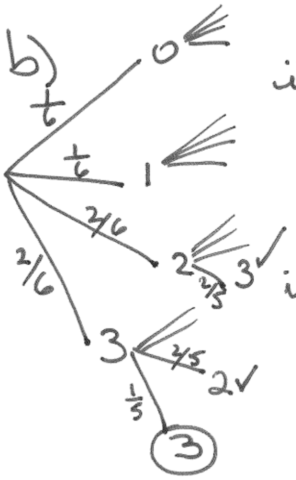
Q14 a) $V = (-3, -2) \quad a = 1$



Focus $(-4,-2)$

direct $x = -2$

b)



i) $P\left(\begin{matrix}Sum \\ 5\end{matrix}\right) = \frac{2}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{2}{5}$

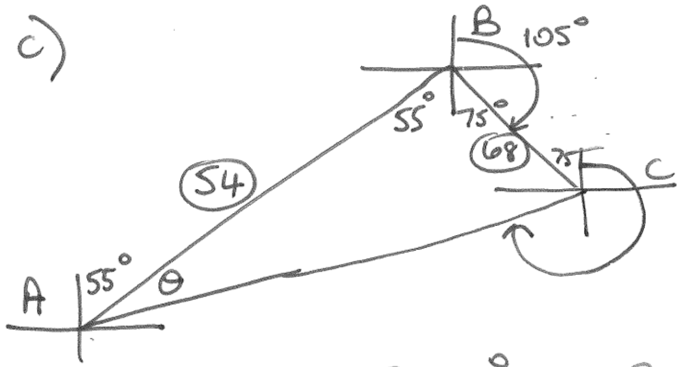
$= \frac{4}{15}$

ii) $P\left(\begin{matrix}Sum \\ <5\end{matrix}\right) = 1 - \frac{4}{15} - \left(\frac{2}{6} \times \frac{1}{5}\right)$

$= 1 - \frac{10}{30}$

$= \frac{20}{30} = \frac{2}{3}$

c)



i) $\angle ABC = 55° + 75° = 130°$

ii) $CA^2 = 54^2 + 68^2 - 2 \times 54 \times 68 \times \cos 130$

$= 12260.63...$

$CA = 110.727 = 110.7 \text{ km}$

iii) $\frac{\sin\theta}{68} = \frac{\sin 130}{110.7}$

$\sin\theta = \frac{68 \sin 130}{110.7} = 0.4705...$

$\theta = 28°$

$\angle C = 180 - 130 - 28 = 22°$

Bearing $= 360 - (75+22)$

$= 263°$

d) $V = \pi \int_0^8 2^2 \, dx - \pi \int_0^8 x^{2/3} \, dx$

$\left[V = \pi \int y^2 \, dx\right]$

$= \pi \int_0^8 4 - x^{2/3} \, dx$

$= \pi \left[4x - \frac{3}{5} x^{5/3}\right]_0^8$

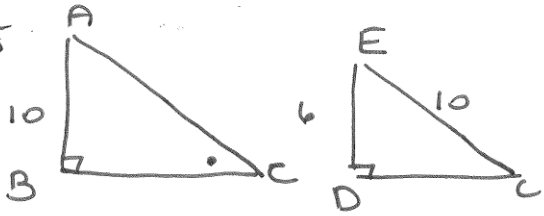
$= \pi \left[32 - \frac{3}{5} \sqrt[3]{8^5} - 0\right]$

$= \pi \left(32 - \frac{3 \times 32}{5}\right)$

$= \frac{64\pi}{5} \, u^3$

Q15

a)



In ΔABC and ΔEDC

$\angle ABC = \angle EDC = 90°$ (given)

$\angle ACB = \angle ECD$ (common $\angle$)

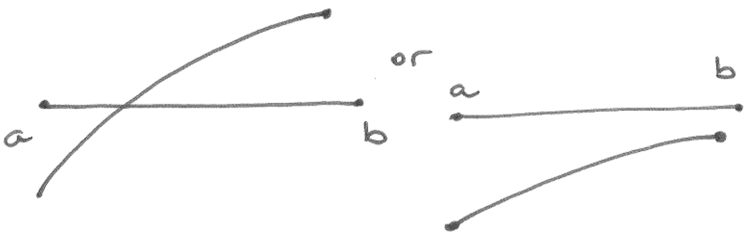
$\therefore \Delta ABC \; ||| \; \Delta EDC$ (equiangular)

ii) $\frac{AC}{EC} = \frac{AB}{ED}$ (ratio of corresp sides in similar Δ's)

$\frac{AC}{10} = \frac{10}{6}$

$AC = \frac{100}{6} = 16\frac{2}{3} \text{ cm}$

Q15b)



c) $3 \quad 3n \quad 3n^2 \ldots\ldots 3n^{10}$ ← 11 terms

$3 \ldots\ldots\ldots 3072$

$3072 = 3n^{10}$

$n^{10} = 1024 \quad \therefore n = 2$

$= 2^{10}$

$3, 6, 12, \ldots\ldots 3072$

$S_{11} = \dfrac{3(2^{11}-1)}{2-1} = 6141$

d) $A = \int_{\pi/3}^{\pi} \sin x \, dx - \int_{\pi/3}^{\pi} \sin 2x \, dx$

$= -\left[\cos x\right]_{\pi/3}^{\pi} + \frac{1}{2}\cos 2x\Big]_{\pi/3}^{\pi}$

$= -(-1-\frac{1}{2}) + \frac{1}{2}(1 - -\frac{1}{2})$

$= 1\frac{1}{2} + \frac{3}{4}$

$= 2.25 \ u^2$

e) i) $AD = 4x\theta$

ii) $ABCD = x + 3x\theta + x$

$= 2x + 3x\theta$

iii) $AD = ABCD$

$4x\theta = 2x + 3x\theta$

$x\theta = 2x$

$x(\theta - 2) = 0$

$\therefore x \neq 0 \quad \theta = 2$ radians

Q16

a) $f''(x) = 6x - 2$

$f'(x) = 3x^2 - 2x + C$

$\frac{dy}{dx} = 0 \quad 0 = 3 - 2 + C$

$C = -1$

$f(a) = 0 = 3a^2 - 2a - 1$

$(3a+1)(a-1) = 0$

$a = -\frac{1}{3}, \ 1$

$\therefore a = -\frac{1}{3}$

b) $2\cos^2 x - 2\sin x \cos x = 0$

$2\cos x(\cos x - \sin x) = 0$

$\cos x = 0 \qquad \cos x = \sin x$

AS $\qquad \tan x = 1$

$x = \pi/2, 3\pi/2 \qquad x = \pi/4, 5\pi/4$

$x = \pi/4, \pi/2, 5\pi/4, 3\pi/2$

c) i) $A_1 = P(1.005) - 232$

$A_2 = [P(1.005) - 232](1.005) - 232$

$= P(1.005)^2 - 232(1.005) - 232$

$= P(1.005)^2 - 232(1 + 1.005)$

ii) $A_n = P(1.005)^n - 232(1 + 1.005 + \ldots 1.005^{n-1})$

$= P(1.005)^n - \dfrac{232 \times 1(1.005^n - 1)}{.005}$

$= P(1.005)^n - 46400(1.005^n - 1)$

iii) $A_{60} = 0$ amt paid back

$\therefore P(1.005)^{60} - 46400(1.005^{60} - 1) = 0$

$P = \dfrac{46400(1.005^{60} - 1)}{1.005^{60}}$

$= \$12000.33$

$\approx \$12000$

Q15d)

$P = n^3 - 30n^2 + 225n$

$\frac{dP}{dn} = 3n^2 - 60n + 225$

$= 3(n^2 - 20n + 75)$ stat pts

$0 = 3(n-5)(n-15)$ $\frac{dP}{dn} = 0$

Stat pts $n = 5, 15$

$\frac{d^2P}{dn^2} = 6n - 60$

$f''(5) = 30 - 60 = -30 < 0$

max value $n = 5$

$f''(15) = 90 - 60 = 30 > 0$

min value $x = 15$

test Endpoints $n = 1.$ $P = 196$

$n = 20$

$P = 20^3 - 30(20)^2 + 225(20)$

$= 500$

$n = 5$ $P = 5^3 - 30(5)^2 + 225(5)$

$= 500$

$\therefore$ Players 5 and 20 have highest score of 500